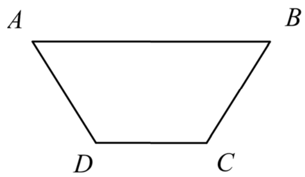
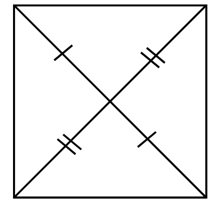
Classify the following are parallelograms, squares, rectangles, rhombi, trapezoids or isosceles trapezoids.

7. assume that $\overline{CD} \parallel \overline{AB}$, $\angle A \cong \angle B$ and $\angle C \cong \angle D$

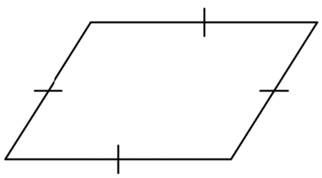


8.

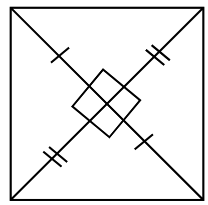
9.



10.

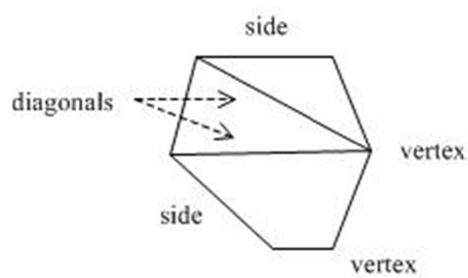


11.



Section 4.4 – Polygons

In Chapter Three we talked about three sided figures; in the previous three sections of this chapter, we talked about four sided figures. Now we're going to take what we've done so far and generalize it to figures with an arbitrary[18] number of sides.

Definition 4.3	A *polygon* is an object made of three or more coplanar segments that intersect only at their endpoints and where each endpoint is shared by exactly two segments. The segments that make up the polygon are called its *sides*. The points where the segments meet are called the polygon's *vertices*. A segment that connects two non-adjacent vertices is called a *diagonal*. The polygon on the right has a total of six sides and six vertices, four of which are labeled. I also drew in two of the nine diagonals. For practice, see if you can draw in the remaining seven. 

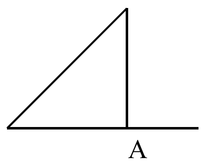
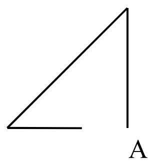
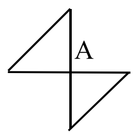
This is kind of like the definition of a triangle: Long and wordy because it's trying to rule out some odd cases that we'll look at below. In plain English, a polygon is a bunch of segments put together in a "closed" figure. We've already seen two examples of this: triangles and quadrilaterals.

In polygons, "side" and "vertex" mean exactly the same thing as they did with triangles. A diagonal means the same thing as it did with quadrilaterals only it can connect any two, non-adjacent vertices. If you think about it for a minute,

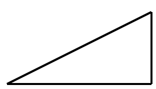
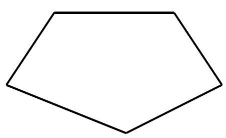
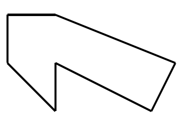
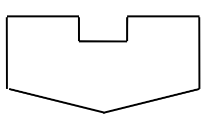
[18] "Arbitrary" is a word mathematicians like. It's kind of a fancy way of saying variable or random. When a mathematician talks about an "arbitrary" number he means it can be anything.

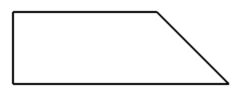
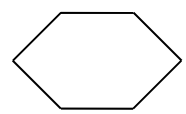
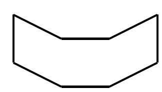
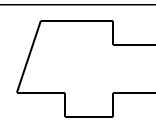
you'll see why we didn't talk about diagonals when we discussed triangles: A triangle doesn't have two vertices that aren't adjacent to each other.

Let's look at some examples of things that aren't polygons so we can see why the definition has to be so long.

	This isn't a polygon because vertex *A* is "shared by" three segments where the definition says there can be at most two.
	This isn't a polygon because vertex *A* is "shared by" only one segment where the definition says it has to be "exactly two".
	This isn't a polygon because two of the segments intersect at a point, *A*, that isn't their endpoint. You could, on the other hand, argue that there are actually two polygons in the diagram: the two triangles.

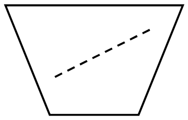
Some polygons have special names depending on the number of sides they have.

Name	Sides	Example
triangle	3	
pentagon	5	
heptagon	7	
nonagon	9	

Name	Sides	Example
quadrilateral	4	
hexagon	6	
octagon	8	
decagon	10	

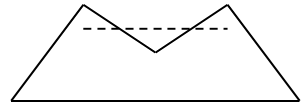
For polygons with more than ten sides we just use the number with –gon added to the end. So a 17 sided polygon would be a 17-gon. In practice you don't need to know all eight of the named polygons. It's usually sufficient if you can recognize a triangle, quadrilateral, pentagon, hexagon and octagon. Those are the ones that show up the most often.

Definition 4.4	A polygon is *convex* if every pair of points inside the polygon is connected by a segment that's also completely inside the polygon. A non-convex polygon is called *concave*.



convex

Pick any two points (I've shown one example) and the segment connecting them stays inside the polygon.



concave

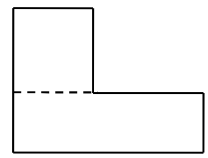
There are pairs of points (I've shown one example) where the segment connecting them leaves the polygon.

If you look back at the named polygons in the table above, the first four (triangle through hexagon) are convex; the last four (heptagon through decagon) are concave.

Area of Polygons

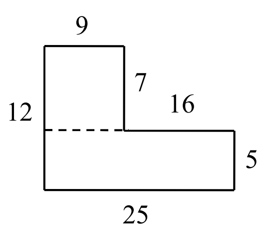
Finding the area of polygons can be a challenge. When we're talking about triangles, we've got several formulas that cover a lot of situations. Quadrilaterals are a little more complicated. We've got some formulas that deal with special cases like parallelograms and trapezoids, but nothing that lets us handle a random quadrilateral. When the number of sides is greater than four there really aren't any formulas to help us out. What we usually do in these cases, is break the polygon into parts that we do have formulas for.

Take a look at the pentagon to the right. If I gave you the lengths of the sides, there's no formula that would let you find the figure's area. However, I can find it by adding the dotted line across the middle. Notice how that line divides the pentagon into two rectangles. We do have a formula for the area of a rectangle so, if I can find the areas of those two pieces, I can find the area of the bigger polygon by adding up the areas of the pieces.



Example 1 – Area of Polygons

Find the area of the polygon to the right.



This is the same polygon that I talked about in the previous paragraph but with some numbers added.

If you look at the bottom rectangle, its dimensions are 5 by 25; if you look at the upper rectangle, its dimensions are 9 by 7. If I use the formula for the area of a rectangle, I see that

$$A_{\text{upper rectangle}} = 9 \cdot 7 = 63$$

$$A_{\text{lower rectangle}} = 5 \cdot 25 = 125$$

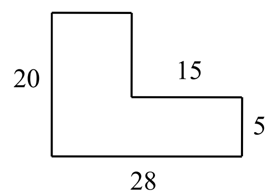
Now, I can get the area of the polygon by adding those pieces together.

$$A_{\text{polygon}} = A_{\text{upper rectangle}} + A_{\text{lower rectangle}}$$

$$A_{\text{polygon}} = 63 + 125 = 188$$

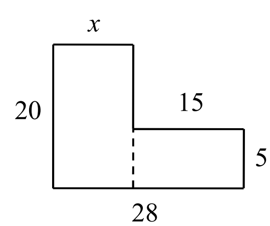
Example 2 – Area of Polygons

Find the area of the polygon to the right.



This question is similar to the previous one but some of the side lengths are missing. No matter where I draw a line to split the figure into rectangles, I'm going to be missing a side. Fortunately, I can calculate what I need using the Segment Addition Postulate.

Suppose I break the polygon up like the figure on the right. I can get the area of the right hand rectangle just by using its dimensions.



$$A_{\text{right rectangle}} = 15 \cdot 5 = 75$$

For the left hand rectangle, I'm missing the length but I can calculate it. Suppose I let it be x. The total length of the polygon is 28 which means I have to have

(continued on next page)

$$x + 15 = 28$$

$$x = 13$$

So the left hand rectangle's dimensions are 20 x 13 which makes its area

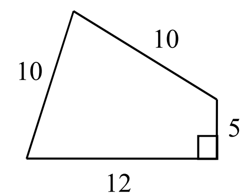
$$A_{\text{left rectangle}} = 20 \cdot 13 = 260$$

Finally, the area of the large polygon has to be

$$A = A_{\text{right rectangle}} + A_{\text{left rectangle}} = 75 + 260 = 335$$

Example 3 – Area of Polygons

Find the area of the polygon to the right.



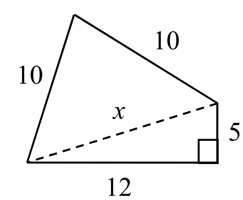
When we're working with a quadrilateral, it's often going to be easier to break the figure up into triangles. In this case, I'm going to draw in a diagonal that connects the bottom left and upper right vertices because of the right angle. That will let me use the Pythagorean Theorem to find the diagonal's length.

$$5^2 + 12^2 = x^2$$

$$25 + 144 = x^2$$

$$x^2 = 169$$

$$x = 13$$



Because the 5 side is an altitude of the bottom triangle, I can find its area using the (1/2)*bh* formula.

$$A_{\text{lower triangle}} = (1/2) \cdot 5 \cdot 12 = 30$$

For the upper triangle, I don't have an altitude but I do have all three side lengths so I'll have to fall back on Heron's formula.

$$s = (10 + 10 + 13) = 33 / 2$$

So the area is

$$A = \sqrt{\frac{33}{2}\left(\frac{33}{2} - 10\right)\left(\frac{33}{2} - 10\right)\left(\frac{33}{2} - 13\right)}$$

$$A = \sqrt{\frac{33}{2} \cdot \frac{13}{2} \cdot \frac{13}{2} \cdot \frac{7}{2}}$$

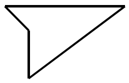
$$A = \sqrt{\frac{13^2 \cdot 33 \cdot 7}{4^2}}$$

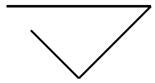
$$A = \frac{13}{4}\sqrt{231}$$

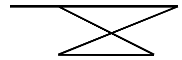
So the area of the whole figure is $A = \frac{13}{4}\sqrt{231} + 30$

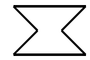
Exercises

Which of the following are polygons? For the ones that aren't explain why.

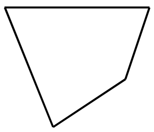
1. 

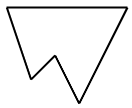
2. 

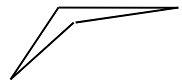
3. 

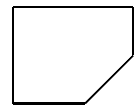
4. 

Label each of the following polygons as convex or concave. If it's concave then show two points where the segment connecting them goes outside the polygon.

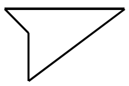
5. 

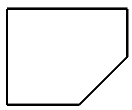
6. 

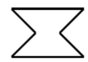
7. 

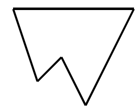
8. 

Label each of the following polygons based on its number of sides.

9. 

10. 

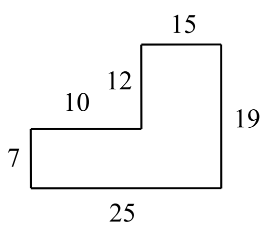
11. 

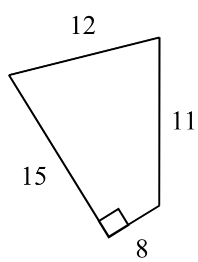
12. 

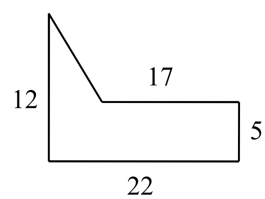
Draw the following polygons.

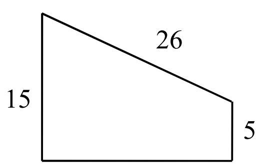
13. a convex hexagon with six congruent sides
14. a concave hexagon with six non-congruent sides
15. a heptagon with non-congruent sides
16. a triangle with congruent sides

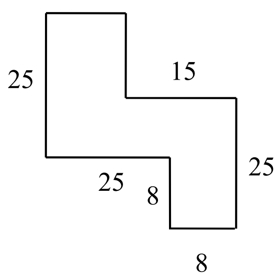
Find the area of the following polygons. You can assume that angles that look like right angles are right angles.

17. 

18. 

19. 

20. 

21. 

Section 4.5 – Angles in Polygons

Back in Chapter 3, we saw that if we took the measures of the angles in a triangle and added them together we always got 180°. It would be great if we could come up with a similar rule for other polygons. It turns out that we can, at least for convex polygons, by building on our rule for triangles.