Chapter 1 – Basic Terms

Section 1.1 – Points, Lines and Planes

In This Section

Everything has to start somewhere. In math, the starting point is usually a set of definitions. In this section, in fact through much of this chapter, we will look at the basic objects of geometry, points, lines, angles, etc., and some of the basic ways that they relate to each other.

Learning Objectives

1. Given a description of a figure using relationships between basic geometric shapes, draw the figure.
2. Provide a descriptive definition of the three undefined terms: point, line and plane.
3. Distinguish between a set of points that are coplanar and a set of points that are not coplanar.
4. Distinguish between a set of points that are collinear and a set of points that are not collinear.
5. Distinguish between a set of lines that are coplanar and a set of lines that are not coplanar.

Required Material

Students don't need any prior knowledge for this section.

Teaching Suggestions

The Real World One of the great things about geometry is that it's the least abstract of all the math classes taught at the high school level. Look around the room and try to find examples of the geometric objects discussed in this section. Points, lines and planes don't actually exist in the real world. Remember that a line, for example, is supposed to go on forever in both directions. What we commonly think of as lines are actually segments. What most people would call a plane, for example a tabletop, is technically a rectangle.

Once you've made this clear you can point out that, while a tabletop isn't a plane, it is contained in one. For convenience, we often use things like tabletops to represent planes. This works because, in real world applications, we only work in a small part of the plane anyway.

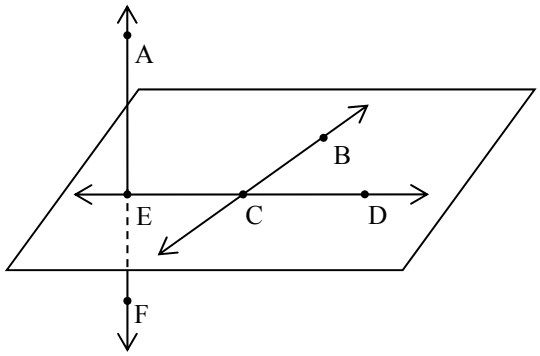
Art Line art is a medium that uses straight and curved lines, usually against a plain background. Although the lines in line art can be curved as well as straight, they are required to have no gradations or color that emphasizes shape and outline. This gives them the same kind of simplicity that our geometric lines have.

Precision Throughout this book, you should always remember that the language of mathematics is extremely precise. When discussing the material you should emphasize the importance of saying what you mean. For example, if you're talking about a segment then don't call it a line and assume the listener will understand what you really meant.

Dimension At this point, you could bring up the concept of dimension. A point is a zero dimensional object, a line is one-dimensional, a plane is two-dimensional and space is three-dimensional. Mathematicians don't stop there. In analysis, for example, it is possible to work in a "space" with any number of dimensions. It's even possible for an object, called a fractal, to have a fractional dimension.

Exercise Solutions

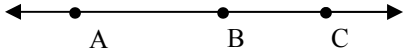
Use the diagram on the right to answer questions 1 through 6. Be sure to use the correct notation.



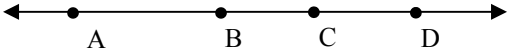
1. **Give the names of four points.** There are six all together: A, B, C, D, E and F.
2. **Give the names of two rays.** There are many possibilities here: $\overrightarrow{AF}$, $\overrightarrow{EF}$, $\overrightarrow{EA}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$ among others. Remember that the order of the letters matters. The ray's endpoint has to be the first letter.
3. **Give the names of two lines.** There are only three: $\overleftrightarrow{ED}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AE}$ although the first and last could be referred to using any of the three points that are on them.
4. **Give the names of two rays that have at least one point in common.** There are several possibilities here as well: $\overrightarrow{CB}$ and either of $\overrightarrow{CD}$ or $\overrightarrow{CE}$ or either of $\overrightarrow{EF}$ and $\overrightarrow{ED}$ and $\overrightarrow{EA}$.
5. **Give the names of two segments with no points in common.** One of $\overline{AE}$ and $\overline{EF}$ and either of $\overline{CB}$ and $\overline{CD}$.
6. **Give the name of a line and a segment contained in it.** $\overleftrightarrow{BC}$ contains $\overline{BC}$, $\overleftrightarrow{EF}$ contains both $\overline{AE}$ and $\overline{EF}$, $\overleftrightarrow{ED}$ contains both $\overline{EC}$ and $\overline{CD}$.

Draw the following.

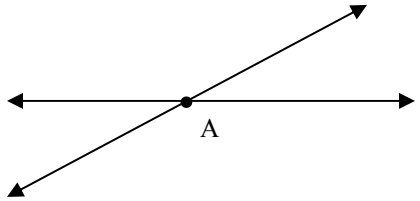
7. **A line and three points on it.**



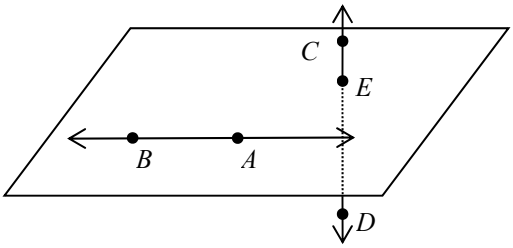
8. **Four points and a line connecting them.**



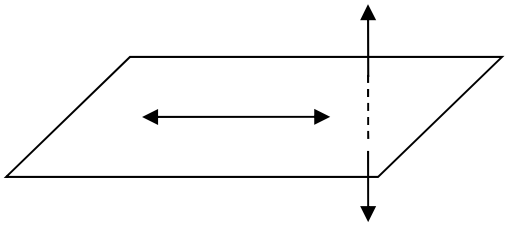
9. **Two lines that cross each other. Label the point where the lines intersect.**



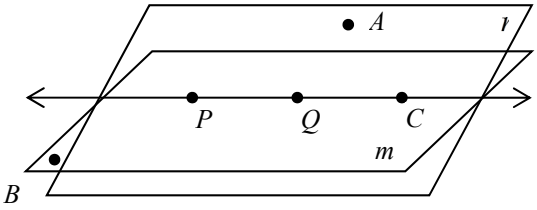
10. **A plane that contains $\overleftrightarrow{AB}$ and that intersects with $\overleftrightarrow{CD}$ at point *E*.**



11. **A plane with one line in it and one line that crosses it.**



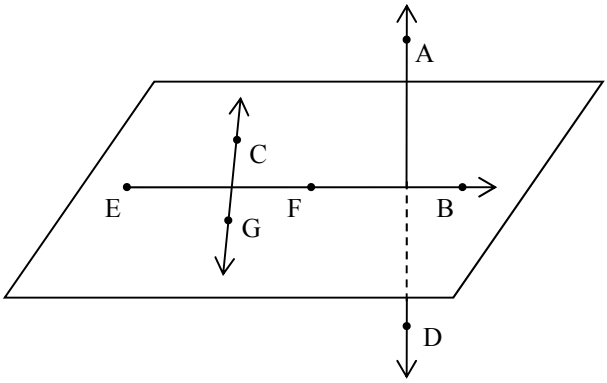
12. **Two planes, *r* and *m*, whose intersection is line $\overleftrightarrow{PQ}$. Put point *A* in plane *r*, point *B* in plane *m* and point C in both planes.**



Indicate whether the following statements are true or false.

13. **Two points are always collinear.** This is true. Pick any two points and you can always draw a line between them.
14. **Three points are always coplanar.** This is also true. Pick any three points and it's always possible to find a plane containing them.
15. **Two lines can never be coplanar.** This one is false. See the answer to question 10 in this section for an example.
16. **Two lines are always coplanar.** This is false. See questions 11 and 12 for examples.

Use the figure to the right to answer questions the following questions.



17. **List three points that are collinear.** *E*, *F* and *B* are collinear. So are *A*, *B* and *D*.
18. **List three points that aren't collinear.** Pick any combination of three points other than the ones in answer 18.
19. **List four points that are coplanar.** Any four of *B*, *C*, *E, F* and *G* or *A*, *D* and any two of *B*, *E* and *F*.
20. **List four points that aren't coplanar.** *A*, *B*, *C* and *D* are one example. In general, you need to pick four points that aren't on a pair of intersecting lines or an intersecting line and ray.
21. **Give two lines that are not coplanar.** $\overleftrightarrow{CG}$ and $\overleftrightarrow{AD}$ are the only ones.