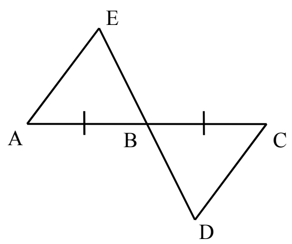
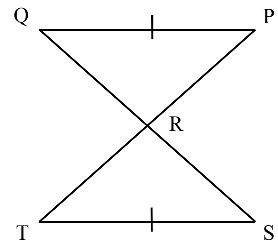
14. **Assume that $\angle A \cong \angle C$.**



We know that $\angle EBA \cong \angle CBD$ because they are vertical angles to the triangles are congruent by the ASA Theorem.

15. **Assume that $\overline{QP} \parallel \overline{TS}$.**



We know that $\angle Q \cong \angle S$ and $\angle P \cong \angle T$ because they are alternate interior angles of a pair of parallel lines. This means the triangles are congruent by the ASA Theorem.

Section 3.4 – The Pythagorean Theorem

In This Section

The Pythagorean Theorem, establishing a relationship between the lengths of the sides of a right triangle, is one of the best known theorems in geometry. In this section, we'll look at the theorem, its converse and some of its applications.

Learning Objectives

1. Solve problems using the Pythagorean Theorem.
2. Use the Converse of the Pythagorean Theorem to determine if a given triangle is a right triangle.

Required Material

Students should be able to recognize a right triangle and solve basic algebraic equations like $3x + 5 = 12$.

Teaching Suggestions

Warm Up You can illustrate the theorem by drawing some random right triangles, measuring their sides and confirming that the sum of the squares of the legs is equal to the square of the hypotenuse. If your students are especially creative, you can encourage them to try to find the relationship on their own but it's not the sort of thing most students will pick up just by looking at the numbers.

Converses "Converse" is a technical term from logic that works like this: If we start with a logical statement, say

If it's raining then I'm inside.

the sentence has two parts that I think of as the "if" part and the "then" part. The technical terms are "antecedent", for the phrase after the "if", and "consequent", for the phrase after the "then". For our example, the antecedent is "it's raining" and the consequent is "I'm inside".

To form the converse of the original statement, you switch the antecedent and the consequent. That means that the converse of, "If it's raining then I'm inside", is, "If I'm inside then it's raining."

Now look at the Pythagorean Theorem: "If a triangle is a right triangle then $a^2 + b^2 = c^2$." The antecedent is, "the triangle is a right triangle" and the consequent is, "$a^2 + b^2 = c^2$". To form the converse, we switch those two parts and end up with, "If $a^2 + b^2 = c^2$ then the triangle is a right triangle."

It's important to keep in mind that just because a statement is true that doesn't automatically mean that its converse is true as well. You can see that with my original example. Even if it's always true that I'm inside when it's raining I could be inside for other reasons so the statement that "If I'm inside then it's raining." won't always be true.

A Short Application – Is An Object Square Suppose you're doing some finish carpentry and you cut a piece of wood to form the edge of a cabinet. It's important that the sides be exact right angles so that they meet up with the other pieces correctly. You could pull out a protractor and try to measure the angle but most carpenters don't carry protractors in their tool belts. An easier way is to apply the converse of the Pythagorean Theorem. To do this measure out a point 3" from a corner along one edge and 4" from the same corner along the other edge. If the distance between the points is 5" then, because $3^2 + 4^2 = 5^2$, you know the triangle formed by the points you measured and the corner is a right triangle so the corner must be the right angle that you needed.

A Shortcut In the student's text, I give a shortcut to determining if a triangle is a right triangle. Specifically, I mention that because the hypotenuse is always the longest side you can just test the one case where the sum of the squares of the shorter sides equals the square of the largest. Why is the hypotenuse the longest side? First, notice that in a right triangle the 90° angle is always the largest. This is because the sum of the angles has to be 180° so the measures of the remaining angles added together must be 90°. This forces both angles to be strictly less than 90°. So if the 90° angle is the biggest angle, Theorem 3.5 tells us that the side opposite it has to be the biggest side.

Pythagorean Triples A Pythagorean triple is a set of three integers that satisfies the equation in the Pythagorean Theorem. For example, (3, 4, 5) is a Pythagorean triple because $3^2 + 4^2 = 5^2$. Similarly, (5, 12, 13) is also, because $5^2 + 12^2 = 13^2$. You should also point out that any multiple of a Pythagorean triple is also a Pythagorean Triple. So, because (3, 4, 5) qualifies so does (6, 8, 10) and (9, 12, 15). This leads us to a problem that puzzled mathematicians for centuries.

Fermat's Last Theorem Pierre de Fermat was a seventeenth century mathematician[8] noted for his work in number theory.[9] Sometime around 1630 Fermat noted in one of his notebooks that there are no positive, integer solutions for the equation $x^n + y^n = z^n$ if $n > 2$. In other words, if we replace the exponent in Pythagoras' equation with any other positive integer then there are no integer "triples" that satisfy it.[10] Fermat scribbled a note in the margin that said, "I have discovered a truly remarkable proof which this margin is too small to contain." Over the next 300 years, this hypothesis, called "Fermat's Last Theorem", became a sort of Holy Grail for mathematicians. Some of the best mathematical minds turned their attention to proving it, and came up empty handed. In 1993, a mathematician named Andrew Wiles gave a three-day lecture at the Isaac Newton Institute in Cambridge where he proved the Shimura-Taniyama-Weil Conjecture – an obscure theorem from an obscure branch of mathematics called "elliptical functions". His conclusion showed, as a special case, that Fermat's Last Theorem was indeed true.

[8] Today, he's considered one of the greatest number theorists ever. In his own time, however, he was a practicing lawyer and only a part-time mathematician.

[9] Number theory is the study of integers and their properties.

[10] Except for the trivial one, (0, 0, 0).

Exercise Solutions

Simplify the following expressions.

1. $\sqrt{\mathbf{49}} = 7$

2. $\sqrt{\mathbf{169}} = 13$

3. $\sqrt{\mathbf{112 - 12}} = \sqrt{100} = 10$

4. $\sqrt{\mathbf{48}} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

5. $\sqrt{\mathbf{360}} = \sqrt{36 \cdot 10} = 6\sqrt{10}$

6. $\sqrt{\mathbf{162}} + \sqrt{\mathbf{72}} = \sqrt{81 \cdot 2} + \sqrt{36 \cdot 2} = 9\sqrt{2} + 6\sqrt{2} = 15\sqrt{2}$

7. $\sqrt{100} - 5\sqrt{4} = 10 - 5 \cdot 2 = 0$

8. $81 + \sqrt{81} = 81 + 9 = 90$

Solve the following equations.

9. $\mathbf{x^2 = 169}$

$x = \sqrt{169} = 13$

10. $\mathbf{x^2 - 144 = 0}$

$x^2 = 144$

$x = \sqrt{144} = 12$

11. $\mathbf{x^2 = 84}$

$x = \sqrt{84}$

$x = \sqrt{4 \cdot 21} = 2\sqrt{21}$

12. $\mathbf{x^2 + 12 = 36}$

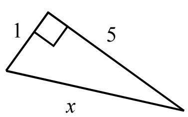
$x^2 = 24$

$x = \sqrt{24}$

$x = \sqrt{4 \cdot 6} = 2\sqrt{6}$

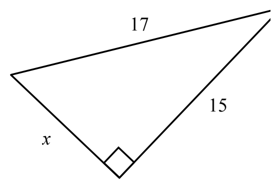
In each triangle, find the missing length.

13.



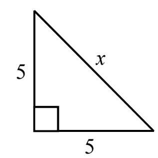
$$x^2 = 1^2 + 5^2 = 26$$
$$x = \sqrt{26}$$

14.



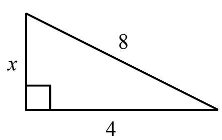
$$x^2 + 15^2 = 17^2$$
$$x^2 = 17^2 - 15^2 = 64$$
$$x = 8$$

15.



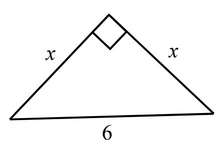
$$x^2 = 5^2 + 5^2 = 50$$
$$x = \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$$

16.



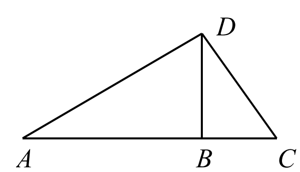
$$x^2 + 4^2 = 8^2$$
$$x^2 = 8^2 - 4^2 = 48$$
$$x = \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

17.



$$x^2 + x^2 = 6^2$$
$$2x^2 = 36$$
$$x^2 = 36 / 2 = 18$$
$$x = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$$

18. **Assume that $\overline{AC} \perp \overline{BD}$, $DB = 5$, $DC = 12$ and $AD = 16$. Find AC.**



$$(AB)^2 + (DB)^2 = (AD)^2$$
$$(AB)^2 + 5^2 = 16^2$$
$$(AB)^2 + 25 = 256$$
$$(AB)^2 = 231$$
$$AB = \sqrt{231}$$

$$(BC)^2 + (DB)^2 = (DC)^2$$
$$(BC)^2 + 5^2 = 12^2$$
$$(BC)^2 + 25 = 144$$
$$(BC)^2 = 119$$
$$BC = \sqrt{119}$$

$$AC = AB + BC = \sqrt{231} + \sqrt{119}$$